CONCLUSIONS

In this paper the ground temperature $T(x,t)$ due to arbitrary solair temperature is expressed in terms of response function and the convolution integral. Analytic solution of the integral is obtained for periodic variation of $T_A(t)$. The resulting expression for $T(x,t)$ is found to be same as that obtained by periodic analysis by earlier authors. The equivalence of two methods for a periodic input is used to determine the preceding significant time in response function method. Subsequently the results of response function analysis and the periodic analysis are compared for a cloudy day preceded by cloudy days.

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BEHAVIOUR OF THE TURBULENT PRANDTL NUMBER NEAR THE WALL

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NOMENCLATURE

Pr,	molecular Prandtl number $\equiv \nu/\gamma$;
Pr_t,	turbulent Prandtl number $\equiv (\overline{uv}\,\partial T/\partial y)/(\overline{v\theta}\,\partial U/\partial y)$;
Q_w,	thermometric wall heat flux;
R_{uv},	correlation coefficient $\overline{uv}/(\overline{u^2}^{1/2}\overline{v^2}^{1/2})$;
$R_{v\theta}$,	correlation coefficient $\overline{v\theta}/(\overline{v^2}^{1/2}\overline{\theta^2}^{1/2})$;
T,	local mean temperature;
T_w,	wall temperature;
T_τ,	friction temperature Q_w/U_τ;
T^+,	$(T_w - T)/T_\tau$;
U, V,	mean velocities in x, y directions, respectively;
U_τ,	friction velocity $= \tau_w^{1/2}$;
U^+,	ratio U/U_τ;
u, v, w,	velocity fluctuations in x, y, z directions, respectively;
u^+, v^+, w^+,	u/U_τ, v/U_τ and w/U_τ, respectively;
$-\overline{uv}$,	Reynolds shear stress;
$\overline{v\theta}$,	turbulent heat flux;
x, y, z,	space co-ordinates in streamwise, normal and spanwise directions;
y^+,	non-dimensional normal co-ordinate yU_τ/ν.

Greek symbols

α_i, β_i,	coefficients in equations (1)–(4);
γ,	thermal diffusivity;
δ_i,	coefficients in equations (11)–(13);
θ,	temperature fluctuation;
θ^+,	θ/T_τ;
τ_w,	kinematic wall shear stress;
ν,	kinematic viscosity.

Subscript

w,	denotes wall value.

INTRODUCTION

THE TREND of Pr_t in the region $0 < y^+ < 40$ and its possible dependence on Pr have not yet been established. Launder [1] suggested that the most sensible requirement is that any proposal of Pr_t in this region should lead to adequate predictions of measured mean temperature profiles and surface heat flux. In this context, Cebeci's [2] model indicates that, close to the wall, Pr_t increases as the wall is approached and remains constant within the viscous sublayer. The constant, as determined by Na and Habib [3] is approximately 1.43. Wassel and Catton [4] use a similar model for their calculation method, except that the constant is about 1.32, again for air. Sleicher [5] calculated Pr_t from measured velocity and temperature profiles of air in fully developed pipe flow and found that Pr_t approached a constant of about 1.4 very near the wall. This value is slightly higher than the value of $Pr^{-1/2}$ suggested, for example, by Sherwood *et al.* [6]. That Pr_t is constant, for a given Pr, very near the wall is verified by analytical considerations of mean velocity, mean temperature, Reynolds shear stress and mean heat-flux profiles in the region close to the wall. Considerations of this type have been given by Meroney [7] and Orlando *et al.* [8]. Meroney did not attempt to estimate the constant, but Orlando *et al.* suggested an experimental procedure that yields a value of about 1.4 for this constant. Although the actual value of Pr_t at the wall is not relevant to methods of calculating the heat transfer in a boundary layer, an accurate description of Pr_t in the buffer zone (approximately $5 < y^+ < 20$) can serve as a useful input to calculation methods. In the present note, the analysis followed in [7] and [8] is used with a view to establish the trend of Pr_t near the wall. This analysis is consistent with the Navier–Stokes and heat-transfer equations and yields a distribution of Pr_t, using available experimental mean velocity, temperature, momentum and heat flux profiles close to the wall.

ANALYSIS AND RESULTS

The behaviour of Pr_t near the wall can be inferred from Taylor series expansions (e.g., [7, 9, 10]) of $\overline{uv}$, $\overline{v\theta}$, U and T in terms of y, viz.,

$$\overline{u^+v^+} = \alpha_1 y^{+3} + \alpha_2 y^{+4} + 0(y^{+5}), \tag{1}$$

$$\overline{v^+\theta^+} = \alpha_3 y^{+3} + \alpha_4 y^{+4} + 0(y^{+5}), \tag{2}$$

$$U^+ = y^+ + \beta_1 y^{+4} + \beta_2 y^{+5} + 0(y^{+6}), \tag{3}$$

$$T^+ = Pr y^+ + \beta_3 y^{+4} + \beta_4 y^{+5} + 0(y^{+6}). \tag{4}$$

The constants $\alpha_i (i = 1\text{–}4)$ can be expressed as correlations at the wall between derivatives, with respect to y, of velocity and/or temperature fluctuations. These constants can be related to the corresponding β_is if the total shear stress $(-\overline{uv} + \nu\,\partial U/\partial y)$ and total heat flux $(\overline{v\theta} - \gamma\,\partial T/\partial y)$ are assumed constant near the wall. The appropriate relations are $\alpha_1 = 4\beta_1$, $\alpha_2 = 5\beta_2$, $\alpha_3 = -4Pr^{-1}\beta_3$, $\alpha_4 = -5Pr^{-1}\beta_4$. At the wall, Pr_t can be written, using equations (1)–(4), as

$$Pr_t = \frac{\alpha_1 Pr}{\alpha_3} = \frac{\beta_1}{\beta_3} Pr^2. \tag{5}$$

This expression, identical to that obtained in [7] (it can also be derived from the results of [9] and [10]), indicates that for a given Pr, Pr_t is constant at the wall. The numerical value of the constant can be obtained from a knowledge of β_1 and β_3.

Values of α_1, α_2, β_1–β_4 may be obtained from least squares regressions of equations (1), (3) and (4) using available experimental data of $\overline{u^+v^+}$, U^+ and T^+ in the wall region. Measurements of $\overline{v^+\theta^+}$ do not seem to be available close to the wall and hence it is not possible to obtain an independent check of the relations between α_3 (or α_4) and β_3 (or β_4). Values of α_i, β_i are shown in Table 1 using the data $(0 < y^+ < 20)$ of Eckelmann [11] who made measurements in a two-dimensional oil channel with a thick viscous sublayer and of Blom [12] who made measurements in a boundary layer downstream of a step in surface temperature. Use was also made of Laufer's [13] data to determine α_1 and α_2. Also shown in the table are α_1 values obtained by Townsend [14] and Coantic [15] who considered the $\overline{u^+v^+}$ data of Laufer [13] and Klebanoff [16]. It is interesting to note that Hinze [17] estimated α_1 to be of order 10^{-4} from a knowledge of measured RMS values of $(\partial u^+/\partial y^+)$ and $(\partial w^+/\partial y^+)$ at the wall and the experimental observation that the spanwise separation z^+ of longitudinal streaks observed close to the wall is about 100. Another way of estimating the βs is to match equations (3) and (4) with the expressions for U^+ and T^+ in the buffer region. These estimates (see Table 1) are in reasonable agreement with those obtained from least squares regressions to the sublayer data. The average values of β_1–β_4 are -1.6×10^{-4}, 5.8×10^{-6}, -1.4×10^{-4} and 5.6×10^{-6} respectively. With these values, equation (5) yields $Pr_t = 0.61$. Near the wall, the behaviour of Pr_t, for air, can be approximated by

$$Pr_t \simeq 0.61 \frac{(1-0.045y^+)}{(1-0.05y^+)}, \tag{6}$$

so that Pr_t has zero slope at the wall and increases sharply near the edge of the sublayer. This is in contrast with the trend in [2, 4, 8] where Pr_t decreases sharply near the edge of the sublayer from a constant value of about 1.4 near the wall. Equation (6) would be consistent with a variation of Pr_t that exhibits a peak near the edge of the sublayer before decreasing toward a constant value of about 0.9 for $y^+ > 40$. Such a trend would be compatible with that suggested by (1) and (18), and in agreement with the Pr_t variation prescribed for one of the calculation methods considered in [19].

A further indirect check of α_1 (or β_1) and α_3 (or β_3) may be obtained by considering the behaviour of the correlation coefficients R_{uv} and $R_{v\theta}$ near the wall. Distributions of $\overline{u^2}^{1/2}$ and $\overline{v^2}^{1/2}$ in the region close to the wall may be written as (e.g. [15], p. 220)

$$\overline{u^{+2}}^{1/2} = \delta_1 y^+ + \delta_2 y^{+2} + 0(y^{+3}), \tag{7}$$

$$\overline{v^{+2}}^{1/2} = \delta_3 y^{+2} + \delta_4 y^{+3} + 0(y^{+4}). \tag{8}$$

It is relatively easy to show that, near the wall, $\overline{\theta^2}^{1/2}$ is given by

$$\overline{\theta^{+2}}^{1/2} = \delta_5 y^+ + \delta_6 y^{+3} + 0(y^{+4}). \tag{9}$$

The coefficients δ_i, inferred from experimental contributions of these RMS quantities close to the wall, are included in Table 1. Using average values of δ_1, δ_3, δ_5 of 0.30, 7.4×10^{-3}, 0.17 and with $\alpha_1 = -5.5 \times 10^{-4}$ and $\alpha_3 = 7.7 \times 10^{-4}$ $(= -4Pr^{-1}\beta_3$ with $Pr = 0.73)$, the correlation coefficients R_{uv} and $R_{v\theta}$ are 0.25 and 0.61, respectively, at the wall. These values do not seem unreasonable in the light of the trend of Fulachier's [21] data for R_{uv} and $R_{v\theta}$ close to the wall.

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Table 1. Coefficients α_i, β_i, δ_i

Investigator	Data	α_1 $(\times 10^4)$	α_2 $(\times 10^5)$	β_1 $(\times 10^4)$	β_2 $(\times 10^6)$	β_3 $(\times 10^4)$	β_4 $(\times 10^6)$	δ_1	δ_3 $(\times 10^3)$	δ_5
Present	[11]	−5.5	2.3	−1.7 (−1.6)†	6.1 (6.4)				6.3	
Present	[12]			−1.4	4.8	−1.4 (−1.4)	5.3 (6)			0.19
Present	[13]								8.6	
Townsend [14]		−6						0.3	9.2	
Coantic [15]	[13, [16]	−11							8	
Elena [20]								0.3		0.15

† Values in brackets are obtained by matching to the buffer region.

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